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Intuitionistic fuzzy gα**-closed sets

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Abstract: *In this paper, we introduce and study the notions of intuitionistic fuzzy gα**-closed sets and intuitionistic fuzzy gα**-open sets and study some of its properties in Intuitionistic fuzzy topological spaces.*

[0]Keywords And Phrases: *Intuitionistic fuzzy topology, Intuitionistic fuzzy gα**-closed sets and Intuitionistic fuzzy gα**-open sets.*

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1 Introduction

In 1965, Zadeh [13] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notions. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce the notions of intuitionistic fuzzy gα**-closed sets and intuitionistic fuzzy g α**-open sets and study some of its properties in intuitionistic fuzzy topological spaces.

2 Preliminaries

Throughout this paper, (X,τ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu A(x), \nu A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFS's of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then
1. $A \subseteq B$ if and only if $\mu_A (X) \leq \mu_B (X)$ and $\nu_A (X) \geq \nu_B (x)$ for all $x \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $A^c = \{ \langle x, \nu_A(x), \mu_A(x)\rangle / x \in X \}$.
4. $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X \}$.
5. $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X \}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1\rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0\rangle / x \in X \}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:
1. $0_\sim, 1_\sim \in \tau$,
2. $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
3. $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then

1. $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A \}$,
2. $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K \}$,
3. $cl(A^c) = (int(A))^c$,
4. $int(A^c) = (cl(A))^c$.

Definition 2.5: [4] An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is said to an

1. intuitionistic fuzzy semi-open set (IFSOS) if $A \subseteq cl(int(A))$,
2. intuitionistic fuzzy pre open set (IFPOS) if $A \subseteq int(cl(A))$,
3. intuitionistic fuzzy α-open set (If α OS) if $A \subseteq int(cl(int(A)))$,
4. intuitionistic fuzzy regular open set (IFROS) if $A = int(cl(A))$,
5. intuitionistic fuzzy β -open set (IF β OS) if $A \subseteq cl(int(cl(A)))$,
6. intuitionistic fuzzy α -open set (IFRαOS) if there exist an IFROS U such that $U \subseteq A \subseteq \alpha cl(U)$.

An IFS A is said to be an intuitionistic fuzzy semi-closed set (IFSCS), intuitionistic fuzzy pre closed set (IFPCS), intuitionistic fuzzy α-closed set (IFαCS), intuitionistic fuzzy regular closed set (IFRCS) and intuitionistic fuzzy β -closed set (IFβCS) if the complement of A is an IFSOS, IFPOS, IFαOS, IFROS and IFβOS respectively.

Definition 2.6: An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is said to an

1. intuitionistic fuzzy generalized closed set (IFGCS) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,
2. intuitionistic fuzzy regular generalized closed set (IFRGCS) [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X,
3. intuitionistic fuzzy generalized semi-closed set (IFGSCS) [7] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,
4. intuitionistic fuzzy α -generalized closed set (IFαGCS) [6] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,
5. intuitionistic fuzzy generalized α -closed set (IFGαCS) [8] if $\alpha cl(A) \subseteq U$whenever $A \subseteq U$ and U is an IFαOS in X,
6. intuitionistic fuzzy generalized semipre closed set (IFGSPCS) [5] if $spcl(A)\subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,
7. intuitionistic fuzzy regular generalized α-closed set(IFRGαCS) [12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFRαOS in X.

An IFS A is said to be an intuitionistic fuzzy generalized open set (IFGOS), intuitionistic fuzzy regular generalized open set (IFRGOS), intuitionistic fuzzy regular generalized α-open set(IFRGαOS), intuitionistic fuzzy generalized semi-open set (IFGSOS), intuitionistic fuzzy α -generalized open set (IFαGOS), intuitionistic fuzzy generalized α-open set (IFGαOS) and intuitionistic fuzzy generalized semipre open set (IFGSPOS) if the complement of A is an IFGCS, IFRGCS, IFRGαCS, IFGSCS, IFαGCS, IFGαCS and IFGSPCS respectively.

Definition 2.7: [12] Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (briefly IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(X) = \begin{cases} (\alpha,\beta), & if\ x = P, \\ (0,1) & otherwise. \end{cases}$$

We observe that an IFP $p_{(\alpha,\beta)}$ is said to belong to an IFS $A = \langle x, \mu_A(x), \nu_A(x) \rangle$, denoted by $p_{(\alpha,\beta)} \in A$ if $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$.

Definition 2.8: [4] Two IFSs A and B are said to be q-coincident ($A\ _q\ B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.9: [4] Two IFSs are said to be not q-coincident ($A\ _q{}^c\ B$ in short) if and only if $A \subseteq B^c$.

3 Intuitionistic Fuzzy G A**-Closed Sets

Definition 3.1: An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy g α**-closed set (briefly IFGα** CS) if $\alpha cl(A) \subseteq int(cl(U))$ whenever $A \subseteq U$ and U is an IFαOS in (X, τ). [i.e.,if $\alpha\ cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in (X, τ)]

Example 3.2: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$. Then the IFS $A = \langle x, (0.4,0.5), (0.6,0.5)\rangle$ is an IFG α **CS in (X, τ).

Theorem 3.3: Every IFCS is an IFGα**CS but not conversely.

Proof: Let $A \subseteq U$, where U is an IFROS. Then $\alpha cl(A) \subseteq cl(A) = A \subseteq U$. Hence A is an IFGα ** CS.

Example 3.4: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle X, (0.5, 0.6), (0.5, 0.4)\rangle$. Then the IFS $A = \langle x, (0.4,0.5), (0.6,0.5)\rangle$ is an IFGα ** CS but not an IFCS in (X, τ).

Theorem 3.5: Every IFRCS is an IFGα** CS but not conversely.

Proof: Since every IFRCS is an IFCS, the proof follows from Theorem 3.3.

Example 3.6: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$. Then the IFS $A = \langle x, (0.4,0.5), (0.6,0.5)\rangle$ is an IFGα** CS but not an IFRCS in (X, τ).

Theorem 3.7. Every IFαCS is an IFGα** CS but not conversely.

Proof: Let A be an IFαCS and U be an IFROS such that $A \subseteq U$. Then $\alpha cl(A) \subseteq U$. Since $\alpha cl(A) = A$ and hence A is an IFGα** CS.

Example 3.8: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$.Then the IFS $A = \langle x, (0.4,0.5), (0.6,0.5)\rangle$ is an IFGα**CS but not an IFαCS in (X, τ).

Theorem 3.9: Every IFGCS is an IFGα** CS but not conversely.

Proof. Let A be an IFGCS and U be an IFROS such that $A \subseteq U$. Since every IFROS is an IFOS and $\alpha cl(A) \subseteq cl(A)$, we have by hypothesis, $\alpha cl(A) \subseteq cl(A) \subseteq U$ and hence A is an IFGα** CS.

Example 3.10. Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$. Then the IFS $A = \langle x, (0.4,0.5), (0.6,0.5)\rangle$ is an IFGα** CS but not an IFGCS in (X, τ).

Theorem 3.11: Every IFRGαCS is an IFGα** CS but not conversely.

Proof: Let A be an IFRG α CS and U be an IFROS such that $A \subseteq U$. Since every IFROS is an IFRαOS and $\alpha cl(A) \subseteq cl(A)$, by hypothesis we have $\alpha cl(A) \subseteq cl(A) \subseteq U$ and hence A is an IFGα** CS.

Example 3.12: Let $X = \{a, b, c\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFTS on X , where $G_1 = \langle x, (0.4, 0.4, 0.5), (0.4, 0.4, 0.4)\rangle$ and $G_2 = \langle x, (0.2, 0.3, 0.5), (0.5, 0.5, 0.5)\rangle$ .Then IFS $A = \langle x, (0.4, 0.3, 0.2), (0.5, 0.4, 0.5)\rangle$ is an IFGα** CS but not an IFRGαCS in (X,τ).

Theorem 3.13: Every IFRGCS is an IFGα** CS but not conversely.

Proof: Let A be an IFRGCS and U be an IFROS such that $A \subseteq U$. Since $\alpha cl(A) \subseteq cl(A)$ and $cl(A) \subseteq U$, By hypothesis, A is an IFGα** CS.

Example 3.14: Let $X = \{a, b, c\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFTS on X, where $G_1 = \langle x,(0.4,0.4, 0.5), (0.4,0.4,0.4)\rangle$ and $G_2 = \langle x, (0.2, 0.3, 0.5), (0.5, 0.5, 0.5)\rangle$. Then the IFS $A= \langle x, (0.4, 0.3, 0.2), (0.5, 0.4, 0.5)\rangle$ is an IFGα** CS but not an IFRGCS in (X, τ).

Theorem 3.15: Every IFαGCS is an IFGα** CS but not conversely.

Proof. Let A be an IFGαGCS and U be an IFROS such that $A \subseteq U$.Since every IFROS is an IFOS and A is IFGαCS, we have $\alpha cl(A) \subseteq U$. Hence A is an IFG α ** CS.

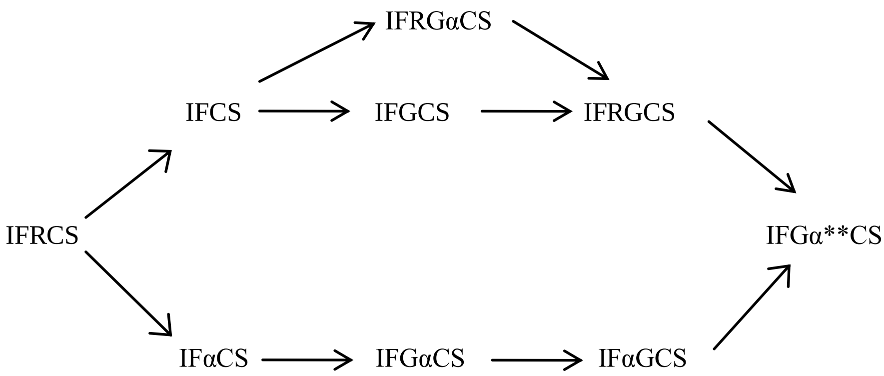
Example 3.16: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle x , (0.5, 0.6), (0.5, 0.4)\rangle$. and Then the IFS $A = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$ is an IFGα** CS but not an IFαGCS in (X, τ).

Theorem 3.17: Every IFGαCS is an IFGα** CS but not conversely.

Proof: Let A be an IFGαCS and U be an IFROS such that $A \subseteq U$. Since every IFROS is an IFαOS and by hypothesis, we have $\alpha cl(A) \subseteq U$. Hence A is an IFGα** CS.

Example 3.18: Let X ={a, b} and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where G =⟨x, (0.8, 0.8), (0.2, 0.1) ⟩. Then the IFS A =⟨x, (0.9, 0.7), (0.1, 0.3) ⟩ is an IFGα** CS but not an IFGαCS in (X, τ).

Remark 3.19: Summing up the above theorems, we have the following diagram. None of the implications are reversible.



Remark 3.20: The following examples show that IFGα** CS is idependent of IFPCS, IFSCS, IFβCS, IFGSCS and IFGSPCS.

Example 3.21: Let X={a, b} and $\tau = \{0_\sim, G1, G2, 1_\sim\}$ be an IFTS on X, where G_1 = ⟨x, (0.7, 0.8), (0.3, 0.2) ⟩ and G_2 = ⟨x, (0.6, 0.7), (0.4, 0.3)⟩.Then the IFS A = ⟨x, (0.6, 0.8), (0.4, 0.2) ⟩ is an IFGα** CS but not an IFPCS, IFSCS, IFβCS, IFGSCS and IFGSPCS in (X, τ).

Example 3.22: Let X={a, b} and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where G =⟨x, (0.5, 0.4), (0.5, 0.6) ⟩. Then the IFS A =⟨x, (0.4, 0.2), (0.6, 0.7) ⟩ is an IFPCS, IFβCS, IFGSCS and IFGSPCS but not an IFG α** CS in (X, τ).

Example 3.23: Let X={a, b} and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where G =⟨x, (0.5, 0.4), (0.5, 0.6) ⟩. Then the IFS A = ⟨x, (0.5, 0.5), (0.5, 0.6) ⟩ is an IFSCS but not an IFGα** CS in (X, τ).

Theorem 3.24: The union of two IFGα** C sets is an IFGα** CS in (X, τ).

Proof: Let U be an IFROS in (X, τ) such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. So, $\alpha cl(A) \subseteq U$ and $\alpha cl(B) \subseteq U$. Therefore $\alpha cl(A) \cup \alpha cl(B) \subseteq \alpha cl(A \cup B) \subseteq U$. Hence $A \cup B$ is an IFGα** CS.

Remark 3.25: Intersection of two IFGα** C sets need not be an IFGα** CS.

Example 3.26: Let X ={a, b, c} and $\tau = \{0_\sim, G_1, G_2, G_1 \cup G_2, G_1 \cap G_2, 1_\sim\}$ be an IFTS on X, where G_1 = ⟨x, (0.0, 0.6, 0.1), (0.1, 0.25, 0.0) ⟩ and G_2 = ⟨x , (0.1, 0.25, 0.0), (0.0, 0.6, 0.1) ⟩. Then the IFS A =⟨x, (0.3, 0.7, 0.1), (0.2, 0.25, 0.3) ⟩ and the IFS B = ⟨x, (0.0, 0.6, 0.3), (0.1, 0.3, 0.0) ⟩ are IFGα** C sets but $A \cap B$ is not an IFGα** CS.

Theorem 3.27: If an IFS A is an IFGα** CS such that $A \subseteq B \subseteq \alpha cl(A)$, where B is an IFS in an IFTS (X, τ), then B is an IFGα** CS in (X, τ).
Proof. Let U be an IFROS in (X,τ) such that $B \subseteq U$. Then $A \subseteq U$. Since A is an IFGα** CS, we have $\alpha cl(A) \subseteq U$. Now, $\alpha cl(B) \subseteq \alpha cl(\alpha cl(A)) = \alpha cl(A) \subseteq U$. Hence B is an IFGα** CS in (X, τ).

Theorem 3.28: If an IFS A is an IFRGCS such that $A \subseteq B \subseteq cl(A)$, where B is an IFS in an IFTS (X,τ), then B is an IFGα** CS in (X, τ).

Proof: Let U be an IFROS in (X,τ) such that $B \subseteq U$. Then $A \subseteq U$. Since A is an IFRGCS and $\alpha cl(A) \subseteq cl(A)$, we have $\alpha cl(A) \subseteq cl(A) \subseteq U$. Now, $\alpha cl(B) \subseteq cl(B) \subseteq cl(A) \subseteq U$. Hence B is an IFGα** CS in (X, τ).

Theorem 3.29: An IFS A is an IFGα** CS in an IFTS (X,τ) if and only if $\rceil(A_qF)$ implies $\rceil(\alpha cl(A)_qF)$ for every IFRCS F of (X, τ).

Proof: Necessity: Assume that A is an IFGα** CS in (X, τ). Let F be an IFRC and $\rceil(A_qF)$. Then $A \subseteq F^c$, where F^c is an IFROS in (X, τ). Then by assumption, $\alpha cl(A) \subseteq F^c$. Hence $\rceil(\alpha cl(A)_qF)$.
Sufficiency: Let F be an IFROS in (X, τ) such that $F \subseteq U$. Then F^c is an IFRCS in (X, τ) and $F \subseteq (U^c)^c$. By assumption, $\rceil(FqU^c)$ implies $\rceil(\alpha cl(A)qU^c)$.Therefore, $\alpha cl(A) \subseteq (U^c)^c = U$. Hence A is an IFGα** CS in (X, τ).

Theorem 3.30: If A is an IFROS and an IFGα** CS in (X,τ). then A is an IFαCS in (X, τ).

Proof: Let A be an IFROS. Since $A \subseteq A$, $\alpha cl(A) \subseteq A$. But $A \subseteq \alpha cl(A)$ always. Therefore $\alpha cl(A) = A$. Hence A is an IFαCS in (X, τ).

Theorem 3.31: Every IFS in an (X, τ) is an IFG α** CS if and only if IFαOS and IFαCS coincide.

Proof: Necessity: Suppose that every IFS in (X,τ) is an IFGα** CS. Let U be an IFROS in (X, τ). Then U is an IFOS and an IFαOS and by hypothesis $\alpha cl(U) \subseteq U \subseteq \alpha cl(U)$. That is $\alpha cl(U) = U$. Thus U is an IFαCS in (X, τ). Hence $IF\alpha O(X) \subseteq IF\alpha C(X)$. Let A be an IFαCS. Then A^c is an IFαOS in (X, τ). But $IF\alpha O(X) \subseteq IF\alpha C(X)$. Therefore A is an IFαOS in (X, τ), we have $IF\alpha C(X) \subseteq IF\alpha O(X)$. Thus $IF\alpha O(X) = IF\alpha C(X)$.
Sufficiency: Suppose that $IF\alpha O(X) = IF\alpha C(X)$. Let $A \subseteq U$ and U be an IFROS in (X, τ). Since every IFROS is IFαOS, U is an IFαOS in (X, τ) and therefore $\alpha cl(A) \subseteq \alpha cl(U) = U$, by hypothesis. Hence A is an IFGα** CS in (X, τ).

Theorem 3.32: An IFS A of an IFTS (X, τ) is an IFROS and an IFGα** CS,then A is an IFRCS in (X, τ).

Proof: Let A be an IFROS and an IFGα** CS in (X, τ). Then $\alpha cl(A) \subseteq A$. Since $\alpha cl(A)$ is an IFαCS, we have $cl(int(cl(A))) \subseteq A$. Therefore $cl(A) \subseteq A$, since A is an IFROS. Then $cl(int(A)) \subseteq cl(A) \subseteq A$. Therefore $cl(int(A)) \subseteq A$. Since every IFROS is an IFSOS, A is an IFSOS and we have $A \subseteq cl(int(A))$. Thus$A = cl(int(A))$. Hence A is an IFRCS in (X, τ).

Theorem 3.33: Let A be an IFGα** CS in (X, τ) and $p_{(\alpha, \beta)}$ be an IFP in X such that $\alpha cl(A)\ _q\ cl(p_{(\alpha, \beta)})$. Then $A\ _q\ \alpha cl(p_{(\alpha, \beta)})$.

Proof: Assume that A is an IFGα** CS in (X,τ) and $\alpha cl(A)\ _q\ cl(p_{(\alpha, \beta)})$: Suppose that $\rceil(A\ _q\ \alpha cl(p_{(\alpha, \beta)})$, then $A \subseteq (\alpha cl(p_{(\alpha, \beta)})^c$ where $(\alpha cl(p_{(\alpha,\beta)}))^c$ is an IFαOS in (X,τ).
Then by Definition 3.1, $\alpha cl(A) \subseteq int(cl(\alpha cl(p_{(\alpha, \beta)}))^c) \subseteq cl((p_{(\alpha, \beta)})^c)$, Therefore $\rceil(\alpha cl(A)\ _q\ cl(p(\alpha, \beta))^c)$, which is a contradiction to the hypothesis. Hence $A\ _q\ \alpha cl(p_{(\alpha, \beta)})$.

4 Intuitionistic Fuzzy Gα** - Open Sets

Definition 4.1: An IFS A of an IFTS (X,τ) is called an IFGα** OS if and only if A^c is an IFGα** CS.

Theorem 4.2: Every IFOS, IFROS, IFαOS, IFGOS, IFRGOS, IFαGOS, IFGαOS is an IFG α** OS in (X, τ).
Proof: Obvious.

Example 4.3: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$.Then the IFS $A = \langle x, (0.6, 0.5), (0.4, 0.5) \rangle$ is an IFGα** OS but not an IFOS, IFROS, IFαOS, IFGOS, IFαGOS in (X, τ).

Example 4.4: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFTS on X, where $G = \langle x, (0.8, 0.8), (0.2, 0.1) \rangle$.Then the IFS $A = \langle x, (0.1, 0.3), (0.9, 0.7) \rangle$ is an IFGα** OS but not an IFGαOS in (X, τ).

Example 4.5: Let $X = \{a, b, c\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFTS on X, where $G_1 = \langle x, (0.4, 0.4, 0.5), (0.4, 0.4, 0.4) \rangle$ and $G_2 = \langle x, (0.2, 0.3, 0.5), (0.5, 0.5, 0.5) \rangle$. Then the IFS $A = \langle x, (0.5, 0.4, 0.5), (0.4, 0.3, 0.2) \rangle$ is an IFGα** CS but not an IFRGαOS, IFRGCS in (X, τ).

Theorem 4.6: An IFS A of an IFTS (X, τ) is an IFGα** OS if and only if $U \subseteq \alpha int(A)$ whenever $U \subseteq A$ and U is an IFRCS.

Proof: Necessity: Assume that A is an IFGα**OS in (X, τ). Let U be an IFRCS such that $U \subseteq A$. Then U^c is an IFROS and $A^c \subseteq U^c$. Then by assumption A^c is an IFGα**CS in (X, τ). Therefore, we have $\alpha cl(Ac) \subseteq U^c$. Hence $U \subseteq \alpha int(A)$.
Sufficiency. Let U be an IFROS in (X, τ) such that $A^c \subseteq U$. Then $U^c \subseteq A$ and U^c is an IFRCS. Therefore $U^c \subseteq \alpha int(A)$. Since $U^c \subseteq \alpha int(A)$. we have $(\alpha int(A))^c \subseteq U$ that is $\alpha cl(A^c) \subseteq U$. Thus Ac is an IFGα**CS. Hence A is an IFGα** OS in (X, τ).

Remark 4.7: Intersection of two IFGα**O sets is an IFGα**OS in $(X; \tau)$: But the union of two IFGα**O sets need not be an IFGα**OS.

Example 4.8: Let $X=\{a,b,c\}$ and $\tau= \{0_\sim, G1, G2, G1 \cup G2, G1 \cap G2, 1_\sim\}$ be an IFTS on X, where $G1 = \langle X, (0.0, 0.6, 0.1), (0.1, 0.25, 0.0) \rangle$ and $G2 = \langle X, (0.2, 0.25, 0.3), (0.3, 0.7, 0.1) \rangle$. Then the IFS $A = \langle X, (0.2, 0.25, 0.3), (0.3, 0.7, 0.1) \rangle$ and the IFS $B\langle X, (0.1, 0.3, 0.0), (0.0, 0.6, 0.3) \rangle$ are IFGα**O sets but $A \cap B$ is not an IFGα**OS.

Theorem 4.9: Let A be an IFS in (X, τ). If B is an IFSOS such that $B \subseteq A \subseteq int(cl(B))$, then A is an IFGOS in (X, τ).

Proof. Since B is an IFSOS, we have $B \subseteq cl(int(B))$. Thus, $A \subseteq int(cl(B)) \subseteq int(cl(cl(int(B)))) = int(cl(int(B))) \subseteq int(cl(int(A)))$. This implies A is an IFαOS. By Theorem 4.2, A is an IFGα** OS in (X, τ).

Theorem 4.10: If an IFS A is an IFGOS in (X, τ) such that $\alpha int(A) \subseteq B \subseteq A$, where B is an IFS in (X, τ), then B is an IFGα** OS in (X, τ):

Proof: Suppose that A is an IFGα** OS in (X, τ) and $int(A) \subseteq B \subseteq A$. Then A^c is an IFGα** CS and $A^c \subseteq B^c \subseteq (\alpha int(A))^c$, this implies $A^c \subseteq B^c \subseteq cl(A^c)$. Then B^c is an IFGα** CS in (X, τ), by Theorem 3. 28. Hence B is an IFGα** OS in (X,τ).

Theorem 4.11: If an IFS A is an IFRGOS in (X, τ) such that $int(A) \subseteq B \subseteq A$,where B is an IFS in (X, τ), then B is an IFG α** OS in (X, τ).

Proof: Let A be an IFRGOS and $int(A) \subseteq B \subseteq A$. Then A^c is an IFRGCS and $A^c \subseteq B^c \subseteq cl(A^c)$. Then B^c is an IFGα** CS in (X,τ), by Theorem 3.28. Hence B is an IFGα**OS in (X, τ).

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